11/14/16 "Quality is not an act, it is a habit" -Aristotle

HW: "Differentiation of Trig Functions" w/s #13-16

AIM: How do we differentiate the other trig functions?

Warm Up:

How can we re-write y = tan(x) ?

$$y = \frac{\sin(x)}{\cos(x)}$$

Differentiation of Trigonometric Functions

If we look at the slope of the tangent lines at the five key points for the sine curve, it can be shown that the derivative is the cosine function. You must memorize these two derivatives.

$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\cos x] = -\sin x$

Let's derive the other four with the help of the trig identity:

$$\sin^2 x + \cos^2 x = 1$$

EX #1: $y = \tan x$

$y = \frac{\sin(x)}{\cos(x)}$

$y' = \frac{\cos(x) \cdot \cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$

$y' = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$

$y' = \frac{1}{\cos^2(x)}$

$\boxed{y' = \sec^2(x)}$

EX #2: $y = \cot x$

$y = \frac{\cos(x)}{\sin(x)}$

$y' = \frac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{\sin^2(x)}$

$y' = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$

$y' = \frac{-1(\sin^2(x) + \cos^2(x))}{\sin^2(x)}$

$y' = \frac{-1}{\sin^2(x)} = \boxed{-\csc^2(x)}$

EX #3: $y = \csc x$

$y = \frac{1}{\sin(x)}$

$y' = \frac{\sin(x) \cdot 0 - 1 \cos(x)}{\sin^2(x)}$

$y' = \frac{-\cos(x)}{\sin^2(x)}$

$y' = \frac{-1}{\sin(x)} \cdot \frac{\cos(x)}{\sin(x)}$

$\boxed{y' = -\csc(x) \cdot \cot(x)}$

EX #4: $y = \sec x$

$y = \frac{1}{\cos(x)}$

$y' = \frac{\cos(x) \cdot 0 - 1(-\sin(x))}{\cos^2(x)}$

$y' = \frac{\sin(x)}{\cos^2(x)}$

$y' = \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)}$

$\boxed{y' = \sec(x) \cdot \tan(x)}$

Derivatives of Trigonometric Functions:

$\frac{d}{dx}[\sin x]=\cos x$	$\frac{d}{dx}[\csc x]=-\csc x \cot x$
$\frac{d}{dx}[\cos x]=-\sin x$	$\frac{d}{dx}[\sec x]=\sec x \tan x$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\frac{d}{dx}[\cot x]=-\csc^2 x$

Now, let's practice! We'll use all the derivative forms we've studied so far: Power, Product, Quotient, and Chain Rule along with trigonometric functions. Here we go. . .

Find the derivative of each function. Simplify, if possible.

EX #5: $y = 4\sin x$

$\boxed{y' = 4\cos(x)}$

EX #6: $y = \sin \underline{4x}$

$y' = \cos(4x) \cdot 4$

$\boxed{y' = 4\cos(4x)}$

EX #7: $y = \sin^2 x$

$y = (\sin(x))^2$

$y' = 2(\sin(x))^1 \cdot \cos(x)$

$\boxed{y' = 2\sin(x) \cdot \cos(x)}$

EX #8: $y = \sin^2(4x)$

$y = (\sin(4x))^2$

$y' = 2(\sin(4x))^1 \cdot \cos(4x) \cdot 4$

$\boxed{y' = 8\sin(4x) \cdot \cos(4x)}$

EX #9: $y = \sin\sqrt{x}$

$y = \sin(x^{1/2})$

$y' = \cos(x^{1/2}) \cdot \frac{1}{2}x^{-1/2}$

$\boxed{y' = \frac{\cos\sqrt{x}}{2\sqrt{x}}}$

EX #10: $y = \sqrt{\sin x}$

$y = (\sin(x))^{1/2}$

$y' = \frac{1}{2}(\sin(x))^{-1/2} \cdot \cos(x)$

$\boxed{y' = \frac{\cos(x)}{2\sqrt{\sin(x)}}}$

EX #11: $y = \sqrt{\sin(4x)}$

$y = (\sin(4x))^{1/2}$

$y' = \frac{1}{2}(\sin(4x))^{-1/2} \cdot \cos(4x) \cdot 4$

$y' = \frac{4\cos(4x)}{2\sqrt{\sin(4x)}} = \boxed{\frac{2\cos(4x)}{\sqrt{\sin(4x)}}}$

EX #12: $y = \sin x \cos x$

$y' = \cos(x)\cos(x) + \sin(x)(-\sin x)$

$\boxed{y' = \cos^2(x) - \sin^2(x)}$

EX #13: $y = x\cos x$

EX #14: $y = \sin x + \cos x$

EX #15: $y = 4\cos x^2$

$y = 4\cos(x^2)$

EX #16: $y = 4\cos^2 x$

$y = 4(\cos(x))^2$

EX #17: $y = \sqrt{\cos 2x}$

EX #18: $y = \tan \sqrt{x}$

EX #19: $y = \tan^2 x$

EX #20: $y = \sec^2 (x - 1)$

EX #21: $y = -4\csc(2x)$

EX #22: $y = \sqrt[3]{\cos x}$

EX #23: $y = \dfrac{x^3}{\cos x}$

EX #24: $y = \sqrt{\tan 3x}$